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FINAL EXAMINATION - M81
JANUARY 1989

PART I. DO ALL EXAMPLES. SHOW ALL WORK. DO ALL PROBLEMS. (4 POINTS EACH)

1. Find the (a) domain (b) range for the function given by $f = \{(x,y) \mid y = 2x^2-4x+3\}$.

2. Evaluate $|x+2[x-4]|$ when $x=-3.72$

3. If $g = \{(x,y) \mid y=\frac{2x-5}{x+3}\}$ find g^{-1} (Inverse of g)

4. Find the remainder when $(5x^{91}-4x^6+2)$ is divided by $(x+1)$

5. What is the multiplicity of $(x-1)$ as a factor of $(x^4-x^3-3x^2+5x-2)$?

6. Write an equation with integral coefficients and of LOWEST possible degree in x having as two of its roots $-2+i\sqrt{2}$ and $-\frac{1}{3}$.

7. Given $6x^7+px^4+qx-5=0$; p & q are integers. Which of the following cannot be a root? (a) -5 (b) $\frac{5}{2}$ (c) $\frac{1}{3}$ (d) $-\frac{1}{4}$ (e) $\frac{5}{6}$ (f) -1

8. Write the quotient $\frac{2\sqrt{3}-4i}{\sqrt{3}+i}$ in standard form.

9. Find the (a) sum (b) product of all the roots of $5x^6=8-3x-2x^3$.

10. Express $(\sqrt{3}+i)^6$ in standard form.

11. Find the INTEGRAL (a) least non-negative upper bound (b) greatest non-positive lower bound of the real roots of $5x^4-8x^2-3x-4=0$.

12. If $f(x)=x^2+2$ and $g(x)=\sqrt{3x-1}$ find (a) $f[g(x)]$ (b) $f[g(6)]$

13. In SIMPLEST FORM find the sixth term in the expansion of $(2x-y^3)^8$.

14. Find the sum of the first fifty terms of the arithmetic progression $-17,-13,-9,-5$

15. Find the sum of the infinite series $\frac{1}{3}-\frac{1}{6}+\frac{1}{12}-\frac{1}{24}+\cdots$.

16. An equation of an ellipse with foci at (0,4) and (0,-4) and whose major axis has a length equal to 10 is:

(a) $\frac{x^2}{25}+\frac{y^2}{16}=1$ (b) $\frac{x^2}{9}+\frac{y^2}{25}=1$ (c) $\frac{x^2}{16}-\frac{y^2}{25}=1$

(d) $\frac{y^2}{25}-\frac{x^2}{16}=1$ (e) none of these.

17. Find the (a) coordinates of the center (b) length of the radius of the circle whose equation is

$x^2+y^2+10x-2y-20=0$.

I. Using the principle of mathematical induction show that $1^2+2^2+3^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$ is true for all positive integers n.

II. Three positive numbers are in an arithmetic progression with common difference of 3. If the first is unchanged, the second increased by 1 and the third is doubled and result increased by 2, then the three new numbers formed are in a geometric progression. Find the original three numbers.

III. Given the complex number 64i

a) Express it in polar form

b) Use DeMoivre's theorem to find its three cube roots in polar form

c) Express these same three roots in standard form.

IV. Given the equation $4x^2-9y^2=36$

$x, x+3, x+6$

$-6 \to 0$

$-6, -3, 0$

$2, 5, 8$

$x, x+4, 2(x+6)+2$

$\frac{(x+4)}{x} = \frac{2x+14}{x+4}$

$(x+4)(x+4) = x(2x+14)$

$x^2+8x+16 = 2x^2+14x$

-6

$x^2+6x-16=0$

$(x+8)(x-2)=0$

$x = -8, +2$

$-8, -4, -2$

$2, 6, 18$

$4 = 2(x+6)+2 \quad x \neq 4$

$4 = 2x+14-x-4$

$4 = x+10$

-6

$-4 \to 12$

$-4, 0,$

$-6, -2, 2$

Name Alex Lozupone
Date 1-6-93

M E 81
Polar Graphs

1 P Convert to polar form

a) $x^2 - 6x + y^2 = 0$

$x^2 + y^2 = 6x$

$r^2 = 6x$

$r = \frac{6x}{r}$

$\boxed{r = 6\cos\theta}$

b) $x - y = 5$

$\frac{x}{r} - \frac{y}{r} = \frac{5}{r}$

$\cos\theta - \sin\theta = \frac{5}{r}$

$r = \cos\theta - \sin\theta$

$\boxed{r = \frac{5}{\cos\theta - \sin\theta}}$

c) $x^2 - y^2 = 4$

$x^2 + y^2 = 4 + 2y^2$

$r^2 = 4 + 2y^2$

$1 = \frac{4}{r^2} + \frac{2y^2}{r^2}$

$1 = \frac{4}{r^2} + 2\sin^2\theta$ $\quad 0 = \frac{4}{r^2} + 2\sin^2\theta - 1$

$0 = \frac{4}{r^2} + \cos 2\theta$

$\frac{4}{r^2} = -\cos 2\theta$

$\boxed{r^2 = 4\cos 2\theta}$

d) $x^2(x^2 + y^2) = y^2$

$x^2 r^2 = y^2$

$xr = y$

$r = \frac{y}{x}$

$\boxed{r = \tan\theta}$

2. Convert to rectangular form

a) $r = 7$

$r^2 = 49$

$\boxed{x^2 + y^2 = 49}$

b) $r = 2 \sin\theta$ $\quad r = 2\frac{y}{r}$

$x^2 + y^2 = 2y$

$\boxed{x^2 + y^2 - 2y = 0}$

c) $r = 2 + 4\cos\theta$

$r = 2 + 4\frac{x}{r}$

$x^2 + y^2 = 2r + 4x$

$\boxed{x^2 + y^2 - 2\sqrt{x^2+y^2} - 4x = 0}$

d) $r = \dfrac{5}{\sin\theta - 3\cos\theta}$

$r = \dfrac{5}{\frac{y}{r} - 3\frac{x}{r}}$ $\quad \cdot r$

$1 = \dfrac{5}{y - 3x}$

$y - 3x = 5$

$\boxed{y = 3x + 5}$

3. Graph: $r = \cos 4\theta$